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Supernova Neutrino Scattering Rates Reduced by Nucleon Spin Fluctuations: Perturbative Limit

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In a nuclear medium, spin-dependent forces cause the nucleon spins to fluctuate with a rate Γ_σ . We have previously shown that as a consequence the effective axial-current neutrino-nucleon scattering cross section is reduced. Here, we calculate this reduction explicitly in the perturbative limit $\Gamma_\sigma \ll T$. By virtue of an exact sum rule of the spin-density structure function, we express the modified cross section in terms of the matrix element for neutrino-nucleon scattering in the presence of a spin-dependent nuclear potential. This representation allows for a direct comparison with and confirmation of Sawyer's related perturbative result. In a supernova core with a typical temperature $T = 10$ MeV, the perturbative limit is relevant for densities $\rho \lesssim 10^{13}$ g cm $^{-3}$ and thus applies around the neutrino sphere. There, the cross-section reduction is of order a few percent and thus not large; however, a new mode of energy transfer between neutrinos and nucleons is enabled which may be important for neutrino spectra formation. We derive an analytic perturbative expression for the rate of energy transfer.

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I. INTRODUCTION

Neutrino scattering rates in a medium differ from those taking place in vacuum. It is well known that spatial correlations between the locations or spins of the target particles can reduce or enhance the average effective scattering cross section. For example, the anticorrelations caused by the Pauli exclusion principle are straightforward to include. Even in a nondegenerate medium, correlations are induced by forces between the targets such as the Coulomb force which thereby causes electromagnetic screening effects [1]. Similarly, in a nuclear medium the spin-dependent nature of the nucleon-nucleon interaction may cause nonnegligible “pairings” of the nucleon spins and thus a reduction of the axial-current neutrino-nucleon scattering rate [2].

We presently study a less familiar cross-section modification which is caused by temporal fluctuations rather than spatial correlations. The main idea is that the

neutrino scattering process takes a certain amount of time. If the energy transfer is ω , the weak probe cannot “resolve” those temporal changes of the target configuration which take place on a time scale faster than about $1/\omega$. For example, the target nucleon spin may flip “during” the neutrino-nucleon collision and thus “cancel itself.” In linear-response theory, this effect is formally described by the frequency dependence of the nucleon dynamical spin-density structure function, which in the relevant limit amounts to the Fourier transform of the autocorrelation function of a single nucleon spin. In the absence of interactions the nuclear spin and thus its autocorrelation function is constant. In the presence of a spin-dependent random force the initial spin direction is forgotten, causing the spin autocorrelation function to decrease to zero for large times. Loosely speaking, then, for small ω (large “duration” of the collision) the weak probe sees a reduced average target spin and thus scatters less efficiently.

A complete treatment should simultaneously include spin fluctuations and spin-spin correlations, and presumably spin waves as well. All these effects are embedded in the dynamical spin-density structure function, which in general has multiple isospin components. However, in contrast to spin-spin correlations, spin fluctuations occur even when there is only one nucleon—provided that its spin is jiggled around. This is a multiple-scattering effect, not a many-body phenomenon. In certain circumstances a pion condensate [3] or the walls in the nuclear bubble phase [4] may be the dominant cause for nucleon spin fluctuations so that it is not entirely academic to study spin fluctuations independently from spin-spin correlations.

Collision-induced changes of particle velocities or spins cause the bremsstrahlung emission of photons, neutrino pairs, or axions. According to the Landau-Pomeranchuk-Migdal (LPM) effect [6,7] the low-energy part of the radiation spectrum is suppressed if multiple interactions destroy the temporal coherence of the source. The spin-fluctuation effects studied here are analogous, except that it is the neutrino scattering rate that is being reduced. While the LPM effect is usually discussed for vector-current processes and thus for velocity fluctuations, in the case of axial-current processes in nonrelativistic nuclear matter the spin fluctuations are more significant. We note that temporal fluctuations do not occur for a conserved quantity such as the charge of a particle. The vectorial nucleon quantity that does fluctuate due to collisions is the velocity, which in the nonrelativistic limit is small. Therefore, in this limit multiple-scattering effects are not important for vector-current neutrino interactions [5]. Still, because in vacuum the nonrelativistic neutral-current neutrino-nucleon scattering cross section is $\sigma = (C_V^2 + 3C_A^2)G_F^2 E_\nu^2 / 4\pi$, any modification of the axial-current part strongly affects the total rate.

The importance of multiple scattering is quantified by the spin fluctuation rate Γ_σ which roughly represents the inverse of the time required for the nucleon to forget its initial spin orientation. This effect is important if Γ_σ is of order the typical energy of the weakly interacting particles which scatter off, or are emitted from, the medium [8], i.e. for $\Gamma_\sigma \gtrsim T$. One can easily estimate (Eq. 15 below) that in a supernova (SN) core with a temperature of order 10 MeV this “high-density case” obtains for $\rho \gtrsim 10^{13} \text{ g cm}^{-3}$. Because densities as large as $10^{15} \text{ g cm}^{-3}$ are encountered in a SN core, quantities like the neutrino opacity or the axion emissivity are impossible to calculate in a purely perturbative way which is based on the assumption that average scattering or emission rates are the incoherent sum of single-scattering events. Interaction rates calculated in the “vacuum limit” are fundamentally flawed for the conditions of a SN core.

To extract meaningful estimates for weak interaction rates one must take recourse to the more general principles of linear-response theory. With our collaborators we have begun to develop this perspective in a series of pa-

pers [5,9–11]. We have argued that the neutrino opacities or axion emissivities can be estimated by virtue of a phenomenological ansatz for the spin-density structure function which incorporates certain limiting cases, notably the low-density one, and which satisfies certain general principles, in particular a sum rule which can be derived independently of perturbation theory. Specifically, we estimated the spin-density structure function for large energy transfers ω using a quasi-bremsstrahlung amplitude (Fig. 1). For small ω , the corresponding neutrino scattering rate diverges as $1/\omega^2$ due to the intermediate nucleon going on-shell. Because the true differential scattering cross section must be finite for all ω , and motivated by considerations of multiple scattering, we advocated replacing $1/\omega^2$ by a Lorentzian $1/(\omega^2 + \Gamma^2/4)$ where Γ is of order Γ_σ , but is adjusted so that the structure function obeys the sum rule.

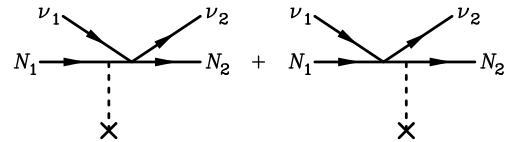


FIG. 1. Neutrino-nucleon scattering in the presence of an external spin-dependent potential for the nucleons. The potential can arise from bystander nucleons, a pion condensate, the walls in the nuclear bubble phase, or some abstract external force.

Meanwhile, Sawyer [12] has published an explicit treatment of the cross-section reduction based on more traditional perturbative techniques. In addition to the quasi-bremsstrahlung graphs of Fig. 1 he includes wavefunction and vertex renormalizations to elastic scattering. The leading correction in the nucleon scattering potential, V , is the interference between zeroth and second order amplitudes, an example of which is shown in Fig. 2. These terms diverge, behaving as $\delta(\omega)$ in the absence of nuclear recoil. However, Sawyer points out, the sum of all order $|V|^2$ contributions yields a total νN cross section which is finite, but reduced from the vacuum value.

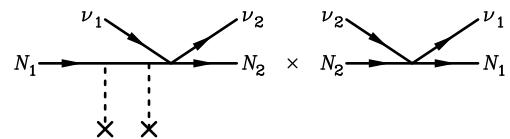


FIG. 2. One diagram representing the interference between zeroth and second order scattering amplitudes that leads to a wave-function renormalization of the incident nucleon for νN scattering.

Motivated by Sawyer’s work, we show how the divergence of the quasi-bremsstrahlung process represented by Fig. 1 can be rigorously controlled by virtue of our exact sum rule of the spin-density structure function without assuming any specific modification of its form, Lorentzian or otherwise, and without calculating the renormalization terms explicitly. However, even though our Lorentzian

ansatz is not needed to obtain the perturbative cross-section reduction effect, it nevertheless yields the correct limiting value because this ansatz incorporates the sum rule explicitly. Put differently, we may implement the sum rule by an explicit ansatz for the low- ω behavior of the spin-density structure function, or we may use the sum rule in an abstract sense. Either way, in the perturbative limit the final result agrees with the one found by Sawyer [12] even though the path of derivation is entirely different. Our novel technique has the added benefit that after the nature of the perturbative region has been understood, the nonperturbative regime may still be studied using our proposed Lorentzian modification or some other related ansatz.

In Sect. II we use the structure-function formalism to derive the perturbative limit of the average axial-current neutrino-nucleon scattering cross section. In Sect. III we consider nucleons interacting with an external classical potential. In this generic example the relationship between the perturbative bremsstrahlung matrix element (Fig. 1) and the cross-section reduction becomes particularly transparent and allows for a direct comparison with Sawyer's [12] result.

In a dilute medium where the perturbative approximation is justified, the most important practical consequence of nucleon spin fluctuations may not be the mild cross-section reduction, but a new mode of energy transfer between neutrinos and the nuclear medium [10]. This energy exchange is enabled by the nontrivial frequency dependence of the spin-density structure function and thus is specific to spin fluctuations; spin-spin correlations do not contribute. Indeed it is plainly visible from the bremsstrahlung nature of the underlying matrix element (Fig. 1) that neutrinos can transfer energy to nucleons above and beyond the standard nucleon recoil effect.

Complementing the numerical expression of Ref. [10] we derive in Sect. IV an analytic expression for the average energy transfer per collision. This perturbative result is relevant for conditions around the neutrino sphere in a SN and thus for the formation of neutrino spectra. Sect. V is given over to discussion and a summary.

II. AVERAGE NEUTRINO SCATTERING RATE

A. Low-Density Limit

The impact of nucleon spin fluctuations on neutrino scattering rates is most easily understood in the long-wavelength limit (see Ref. [5] for a discussion) which has been employed in virtually all previous papers dealing with neutrino opacities, or neutrino pair and axion emissivities, in SN cores or old neutron stars. In this limit, the momentum transfer between neutrinos and nucleons is neglected. The axial-current scattering cross section may then be written as

$$\frac{d\sigma_A}{d\varepsilon_2} = \frac{3C_A^2 G_F^2}{4\pi} \frac{\varepsilon_2^2 S(\varepsilon_1 - \varepsilon_2)}{2\pi}, \quad (1)$$

where ε_1 and ε_2 are the initial- and final-state neutrino energies, G_F is the Fermi constant, and the neutral-current axial weak coupling constant in a dilute medium is $C_A \approx +1.37$ and -1.15 for protons and neutrons, respectively [5].

For simplicity we focus on an isotropic, nonrelativistic, nondegenerate medium of baryon density n_B , temperature T , and a single species of nucleons. In this case the function $S(\omega)$ is the dynamical spin-density structure function in the $\mathbf{k} \rightarrow 0$ limit [5,13]

$$S(\omega) = \frac{4}{3n_B} \int_{-\infty}^{+\infty} dt e^{i\omega t} \langle \boldsymbol{\sigma}(t) \cdot \boldsymbol{\sigma}(0) \rangle, \quad (2)$$

where $\langle \boldsymbol{\sigma}(t) \cdot \boldsymbol{\sigma}(0) \rangle$ is the autocorrelation function for the nucleon spin operator $\boldsymbol{\sigma}(t) = \int d^3 \mathbf{x} \boldsymbol{\sigma}(x)$ at time t . Here $\boldsymbol{\sigma}(x) \equiv \frac{1}{2}\psi^\dagger(x)\boldsymbol{\tau}\psi(x)$, $\psi(x)$ is the nucleon field (a Pauli two-spinor) and $\boldsymbol{\tau}$ is a vector of Pauli matrices. The expectation value $\langle \dots \rangle$ is taken over a thermal ensemble so that detailed balance $S(\omega) = S(-\omega) e^{\omega/T}$ is satisfied. We note that our definition of energy transfer is positive for energy given to the medium.

In order to derive an average scattering cross section we consider nondegenerate thermal neutrinos which we take to follow a Maxwell-Boltzmann distribution; the difference to a Fermi-Dirac distribution is inessential for the present discussion. Therefore, we consider the quantity

$$\langle \sigma_A \rangle \equiv \frac{3C_A^2 G_F^2}{4\pi} \frac{\int d^3 \mathbf{k}_1 e^{-\varepsilon_1/T} \int_0^\infty d\varepsilon_2 \varepsilon_2^2 S(\varepsilon_1 - \varepsilon_2)}{2\pi \int d^3 \mathbf{k}_1 e^{-\varepsilon_1/T}}. \quad (3)$$

With the dimensionless energy transfer $x \equiv (\varepsilon_1 - \varepsilon_2)/T$ and after one explicit integration one finds [5]

$$\langle \sigma_A \rangle = \sigma_T \int_0^\infty \frac{dx}{2\pi} \tilde{S}(x) (2 + x + \frac{1}{6}x^2) e^{-x}. \quad (4)$$

Here, $\sigma_T \equiv \frac{9}{\pi} C_A^2 G_F^2 T^2$ while $\tilde{S}(x) \equiv T S(xT)$ is the dimensionless structure function. In vacuum the nucleon spins do not evolve, yielding a constant autocorrelation function and thus $\tilde{S}(x) = 2\pi\delta(x)$. Then $\langle \sigma_A \rangle = \sigma_T$ where $\int_0^\infty dx \delta(x) = \frac{1}{2}$ has been used.

For reasons that will soon become apparent, we concentrate not on a direct calculation of the average cross-section at finite density $\langle \sigma_A \rangle$, but rather on its deviation from the vacuum cross section $\delta\langle \sigma_A \rangle \equiv \langle \sigma_A \rangle - \sigma_T$ or

$$\frac{\delta\langle \sigma_A \rangle}{\sigma_T} = -1 + \int_0^\infty \frac{dx}{2\pi} \tilde{S}(x) (2 + x + \frac{1}{6}x^2) e^{-x}. \quad (5)$$

The crucial step is to express the r.h.s. as a common integral over $\tilde{S}(x)$. To this end we use the normalization $\int_{-\infty}^{+\infty} \tilde{S}(x) dx / 2\pi = 1$ which obtains if the spins of different nucleons evolve independently; otherwise an additional correlation term would appear on the r.h.s.

[5,10,11], a possibility to be addressed in Sect. II.B below. By virtue of detailed balance this sum rule is

$$\int_0^\infty \frac{dx}{2\pi} \tilde{S}(x) (1 + e^{-x}) = 1. \quad (6)$$

Replacing -1 in Eq. (5) by the negative of Eq. (6) yields

$$\frac{\delta\langle\sigma_A\rangle}{\sigma_T} = - \int_0^\infty \frac{dx}{2\pi} \tilde{S}(x) G(x), \quad (7)$$

where $G(x) \equiv -[(2 + x + \frac{1}{6}x^2)e^{-x} - (1 + e^{-x})]$ or

$$G(x) = 1 - (1 + x + \frac{1}{6}x^2)e^{-x}. \quad (8)$$

This function is shown in Fig. 3. It expands as $G(x) = \frac{1}{3}x^2 + \mathcal{O}(x^3)$ for small x , approaches 1 for large x , and is always positive. Because $\tilde{S}(x)$ is also a positive function, we find that the average cross section in the medium is indeed always suppressed by spin fluctuations.

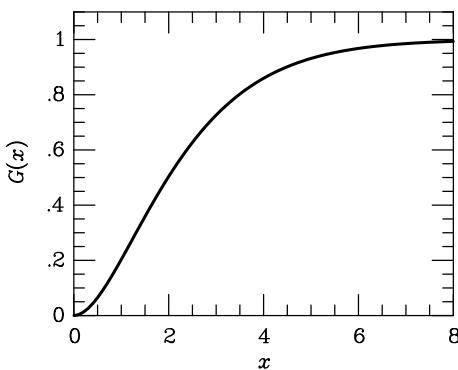


FIG. 3. Function $G(x)$ as defined in Eq. (8).

Thus far $S(\omega)$ has been the nonperturbative but unknown structure function. However, what can be calculated in the framework of perturbation theory is an expression $S_{\text{brems}}(\omega)$ based on the “bremsstrahlung” or “medium excitation” amplitude Fig. 1. In our previous works [5,8], we showed that $S_{\text{brems}}(\omega)$ diverges for small ω as ω^{-2} , a behavior which is generic for all bremsstrahlung processes—for the electromagnetic case see Jackson [14]. We may then write

$$S_{\text{brems}}(\omega) = \frac{\Gamma_\sigma}{\omega^2} s(\omega/T) \times \begin{cases} e^{\omega/T} & \text{for } \omega < 0, \\ 1 & \text{for } \omega > 0, \end{cases} \quad (9)$$

where $s(x)$ is a nonsingular even function with $s(0) = 1$. The quantity Γ_σ , defined as the coefficient of the ω^{-2} singularity of $S_{\text{brems}}(\omega)$, is physically interpreted as the spin-fluctuation rate.*

It should now be clear why we have calculated the deviation $\delta\langle\sigma_A\rangle$ rather than $\langle\sigma_A\rangle$ itself—the x^2 behavior of $G(x)$ compensates for the singularity in S_{brems} . We are therefore free to substitute $S_{\text{brems}}(\omega)$ for $S(\omega)$ and are assured of a finite answer for $\delta\langle\sigma_A\rangle$. Further, if we accept that in a dilute medium the true $S(\omega)$ is well-represented by $S_{\text{brems}}(\omega)$ for $\omega \gg \Gamma_\sigma$ then

$$\frac{\delta\langle\sigma_A\rangle}{\sigma_T} = - \int_0^\infty \frac{dx}{2\pi} \tilde{S}_{\text{brems}}(x) G(x) \quad (10)$$

is the desired perturbative result.

With our representation Eq. (9) the cross-section reduction is to lowest order in $\gamma_\sigma \equiv \Gamma_\sigma/T$

$$\frac{\delta\langle\sigma_A\rangle}{\sigma_T} = - \frac{\gamma_\sigma}{2\pi} \int_0^\infty dx \frac{G(x)}{x^2} s(x). \quad (11)$$

Taking for simplicity the classical limit $s(x) = 1$ we find

$$\frac{\delta\langle\sigma_A\rangle}{\sigma_T} = - \frac{5}{6} \frac{\gamma_\sigma}{2\pi}. \quad (12)$$

Once more, these results put in evidence that γ_σ is the expansion parameter which defines the perturbative regime.

We may estimate the error due to using $\tilde{S}_{\text{brems}}(x)$ in Eq. (10) instead of the full $\tilde{S}(x)$ in Eq. (7). If the true $\tilde{S}(x)$ is given by $\tilde{S}_{\text{brems}}(x)$ to lowest order in γ_σ so that $\tilde{S}(x) - \tilde{S}_{\text{brems}}(x) = \mathcal{O}(\gamma_\sigma^2)$ for $x \gg \gamma_\sigma$, then $\int_0^\infty [\tilde{S}(x) - \tilde{S}_{\text{brems}}(x)] G(x) dx / 2\pi = \mathcal{O}(\gamma_\sigma^2)$.

This implies that the lowest-order cross-section reduction effect represented by Eq. (11) will be found by any assumed functional form $S_{\text{approx}}(\omega)$ for the true $S(\omega)$ if $S_{\text{approx}}(\omega)$ agrees with $S_{\text{brems}}(\omega)$ to $\mathcal{O}(\gamma_\sigma^2)$ for $\omega \gg \Gamma_\sigma$. Any such function which is normalized can be inserted into Eq. (4) and will then yield Eq. (11) up to an error of $\mathcal{O}(\gamma_\sigma^2)$. Further, any such function, even if it is not normalized, will yield this result when inserted into Eq. (7) where the normalization condition has been reshuffled into the function $G(x)$. To lowest order in γ_σ , the cross-section reduction effect is independent of the detailed structure of the true $S(\omega)$ in the neighborhood of $\omega = 0$.

abruptly by a random amount Δs in a given collision (which is thus assumed to be “hard”) and if subsequent spin orientations are uncorrelated one finds $S_{\text{brems}}(\omega) = \Gamma_\sigma / (\omega^2 + \Gamma_\sigma^2/4)$ with $\Gamma_\sigma = \Gamma_{\text{coll}} \langle (\Delta s)^2 \rangle / \langle s^2 \rangle$ [7,15]. This justifies identifying Γ_σ with an average spin rate of change or a spin-fluctuation rate. In the classical limit of hard collisions one has $s(x) = 1$, while for general interaction potentials $s(x)$ is more complicated. Quantum corrections introduce the detailed-balance factor, and cause $s(x)$ to be a decreasing function for large x , as discussed for the case of electromagnetic bremsstrahlung by Jackson [14]. The same conclusion is inferred from the f-sum rule for $S(\omega)$ [11].

*One may consider the limit of a classical spin vector $\mathbf{s}(t)$ being kicked by a random force at a rate Γ_{coll} . If the spin changes

B. Spin-Spin Correlations

A crucial step in the above analysis was use of the sum rule in Eq. (6), appropriate for a medium of uncorrelated nucleons. However, in a real nuclear medium the nucleon spin fluctuations are typically caused by a spin-dependent interaction among nucleons. Inevitably, this will cause correlations between different spins so that the r.h.s. of the sum rule Eq. (6) is $1 + C(\gamma_\sigma)$ where in a dilute medium $|C(\gamma_\sigma)| \ll 1$. It follows that $G(x)$ receives an additional contribution $-C(\gamma_\sigma)(1 + e^{-x})$ and $\delta\langle\sigma_A\rangle/\sigma_T$ one of order $C(\gamma_\sigma)$. Here $\delta\langle\sigma_A\rangle/\sigma_T$ is to be calculated with the full $\tilde{S}(x)$ not $\tilde{S}_{\text{brems}}(x)$. If $C(\gamma_\sigma)$ is of order γ_σ^n , then the correction to the cross-section shift from considering spin-correlations is also of order γ_σ^n .

We may next use the above estimate of the error incurred by using $\tilde{S}_{\text{brems}}(x)$ rather than the true $\tilde{S}(x)$. Then, if $C(\gamma_\sigma) \propto \gamma_\sigma^n$, with $n > 1$ the cross-section deviation calculated from $\tilde{S}_{\text{brems}}(x)$ in Eq. (10) is to lowest order independent of spatial spin-spin correlations.

For example, if the nucleon-nucleon interaction potential is written as in Ref. [11], and if the correlation length scales as γ_σ , then we expect $C(\gamma_\sigma)$ to be of order γ_σ^3 , in which case the low-density limit of the cross-section change is well described by Eq. (10).

C. Comparison with the High-Density Behavior

We next compare the low-density limit thus derived with our more general previous expression. In a dense medium ($\gamma_\sigma \gtrsim 1$) the detailed structure of $S(\omega)$ for low energy transfers matters. In the past we have advocated a Lorentzian form

$$S_{\text{approx}}(\omega) = \frac{\Gamma_\sigma}{\omega^2 + \Gamma^2/4} s(\omega/T) \times \begin{cases} e^{\omega/T} & \text{for } \omega < 0, \\ 1 & \text{for } \omega > 0, \end{cases} \quad (13)$$

where for a given Γ_σ one chooses Γ such that $S_{\text{approx}}(\omega)$ is normalized. This ansatz is motivated by a heuristic argument [8] and by the classical limit which obtains for $\omega \ll T$ [7,15]. Equation (13) naturally approaches the appropriate limit for low densities.

In Fig. 4 we show $\langle\sigma_A\rangle/\sigma_T$ for $s(x) = 1$ as a function of γ_σ . The dotted line marks the “naive” constant cross section which obtains when spin fluctuations are ignored entirely. The dashed line represents the perturbative result according to Eq. (12); for $\gamma_\sigma \gtrsim 7.5$ it yields complete nonsense (a negative scattering cross section). The solid line marked “Lorentzian” was obtained with the above ansatz for $S_{\text{approx}}(\omega)$. The dashed line is its tangent at the point $\gamma_\sigma = 0$ so that indeed the Lorentzian ansatz yields the same perturbative limit as the direct calculation in Sect. II.A where the sum rule was implemented in an abstract sense rather than by a specific ansatz for the low- ω behavior of $S(\omega)$. The Lorentzian ansatz yields

a plausible intermediate result between the naive and lowest-order perturbative results.

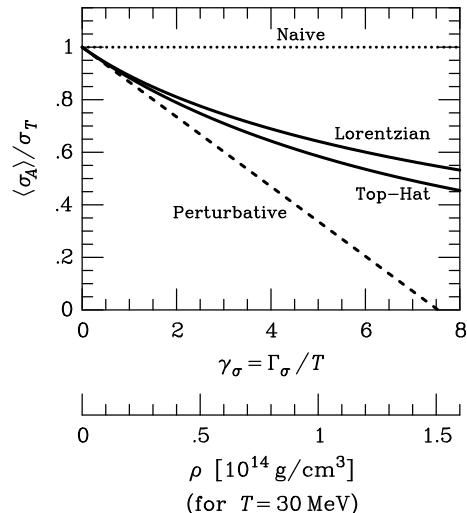


FIG. 4. Average axial-current neutrino-nucleon scattering cross section as a function of the spin-fluctuation rate in the classical limit where $s(x) = 1$. The dotted line refers to a naive calculation where nucleon spin fluctuations are ignored entirely. The dashed line refers to the low-density perturbative expansion which agrees with Sawyer’s [12] related result if one expresses the spin-fluctuation rate Γ_σ in terms of the assumed nucleon interaction potential (Sect. III.A). The solid line marked “Lorentzian” arises from the ansatz Eq. (13) while the one marked “Top-Hat” arises from Eq. (14). The second horizontal axis expresses γ_σ in terms of the physical density ρ for $T = 30 \text{ MeV}$, using the OPE interaction potential according to Eq. (15). The correspondence between γ_σ and ρ is meaningful only in the perturbative regime where $\gamma_\sigma < \text{a few}$.

The overall shape of the Lorentzian line in Fig. 4 is determined by the bremsstrahlung wings of $S_{\text{approx}}(\omega)$ together with the sum rule. In order to test how sensitive it is to the assumed low- ω shape we have considered a second ansatz of the form

$$S_{\text{approx}}(\omega) = \Gamma_\sigma \times \begin{cases} \omega_0^{-2} & \text{for } 0 \leq \omega \leq \omega_0, \\ \omega^{-2} & \text{for } \omega > \omega_0. \end{cases} \quad (14)$$

Of course, for $\omega < 0$ we have the detailed-balance factor $e^{\omega/T}$ as in Eq. (13), and in general there is a function $s(\omega/T)$ which we take to be equal to 1 for the purpose of illustration. For a given choice of Γ_σ the frequency ω_0 is determined such that $S_{\text{approx}}(\omega)$ fulfills the sum rule. The cross-section reduction derived from this “top-hat” ansatz is shown in Fig. 4. It has a common tangent at $\gamma_\sigma = 0$ with the dashed line and the Lorentzian curve, again confirming that in the perturbative limit the detailed low- ω shape of $S(\omega)$ does not matter. For large γ_σ the deviation from the Lorentzian curve is relatively small. Therefore, it appears that even in the nonperturbative regime the cross-section reduction is dominated by the bremsstrahlung calculation in conjunction with

the sum-rule, and not by fine points of the low- ω shape of $S(\omega)$.

In this discussion we have used the classical bremsstrahlung limit of hard collisions where $s(x) = 1$. Quantum corrections alone require that $s(x)$ is a decreasing function of x for large x , and the same conclusion is reached on the basis of Sigl's f-sum rule [11]. Further, the detailed large- x behavior depends on the short-distance behavior of the assumed NN interaction potential. It is evident from Eq. (11) that the detailed functional form of $s(x)$ will determine the slope of the curves in Fig. 4 at $\gamma_\sigma = 0$. However, any change in slope is independent of the manner by which $\tilde{S}(x)$ has been adjusted to satisfy the sum rule. Thus, the three curves remain tangent to each other, although their slope will in general be different from the value $-\frac{5}{6} \frac{1}{2\pi}$ determined for $s(x) = 1$.

To express the spin-fluctuation rate in terms of the physical density and temperature we need to assume a specific model for the cause of the spin fluctuations. Taking nucleon-nucleon interactions modelled by a one-pion exchange (OPE) potential as in our previous papers one finds for a single species of nucleons

$$\Gamma_{\sigma,\text{OPE}} = 4\sqrt{\pi} \alpha_\pi^2 \frac{n_B T^{1/2}}{m_N^{5/2}} = 8.6 \text{ MeV} \rho_{13} T_{10}^{1/2}, \quad (15)$$

where $\alpha_\pi \equiv (f^2 m_N/m_\pi)^2/4\pi \approx 15$ with $f \approx 1.0$ is the pion fine structure constant, $\rho_{13} \equiv \rho/10^{13} \text{ g cm}^{-3}$, $T_{10} \equiv T/10 \text{ MeV}$, and $m_N = 940 \text{ MeV}$ is the nucleon mass. The pion mass has been neglected. Taking $T \approx 10 \text{ MeV}$ as a typical value for SN conditions, one concludes that the dividing line between high and low density is roughly given by $10^{13} \text{ g cm}^{-3}$ or 3% nuclear density.

We stress that $\Gamma_{\sigma,\text{OPE}}$ is in itself a perturbative result and thus will be a reasonable representation of the true Γ_σ only if $\Gamma_{\sigma,\text{OPE}}/T \lesssim 1$. Therefore, in the high-density regime Eq. (15) cannot be used to translate an assumed spin-fluctuation rate into a corresponding physical density. We have previously argued that the true γ_σ in a nuclear medium never exceeds a few [10,11].

III. MATRIX-ELEMENT REPRESENTATION

A. Perturbative Cross-Section Reduction

The perturbative structure function $S_{\text{brems}}(\omega)$ is calculated from the quasi-bremsstrahlung process shown in Fig. 1 so that one may represent the cross-section reduction $\delta\langle\sigma_A\rangle$ directly in terms of its matrix element \mathcal{M} . The translation is most easily achieved by considering the differential scattering cross section. Denoting the four-momentum of the in- and outgoing nucleon with (E_1, \mathbf{p}_1) and (E_2, \mathbf{p}_2) , respectively, we find

$$\begin{aligned} d\sigma_A &= \frac{n_c}{n_B} \varepsilon_2^2 \int \frac{d\Omega_2}{(2\pi)^3} \frac{d^3\mathbf{p}_1}{(2\pi)^3} \frac{d^3\mathbf{p}_2}{(2\pi)^3} \frac{d^3\mathbf{k}}{(2\pi)^3} f_1 \\ &\times \sum_{\text{spins}} \frac{\langle|\mathcal{M}|^2\rangle}{2\varepsilon_1 2\varepsilon_2 2E_1 2E_2} (2\pi)^3 \delta^3(\mathbf{p}_1 + \mathbf{k} - \mathbf{p}_2) \\ &\times 2\pi \delta(E_1 + \varepsilon_1 - E_2 - \varepsilon_2). \end{aligned} \quad (16)$$

Here, n_c is the number density of classical scattering centers, f_1 is the occupation number of the initial-state nucleon, \mathbf{k} is the momentum absorbed by the external potential, Pauli blocking factors are ignored for all particles because of the assumed nondegeneracy, and the neutrinos have been ignored in the momentum δ function because of the long-wavelength approximation. The expectation value $\langle|\mathcal{M}|^2\rangle$ is understood to include the averaging of classical ensemble variables on which the external potential might depend. The perturbative structure function $S_{\text{brems}}(\omega)$ is obtained by comparing Eq. (16) with Eq. (1).

In order to derive the matrix element we use the axial part of the weak interaction Hamiltonian and an external classical potential for the nucleon spins. The most general form for the potential in Fourier space is [16]

$$\begin{aligned} V(\mathbf{k}, \boldsymbol{\sigma}, \mathbf{s}) &= U_0(k) + U_S(k) \boldsymbol{\sigma} \cdot \mathbf{s} \\ &+ U_T(k) (3\boldsymbol{\sigma} \cdot \hat{\mathbf{k}} \mathbf{s} \cdot \hat{\mathbf{k}} - \boldsymbol{\sigma} \cdot \mathbf{s}), \end{aligned} \quad (17)$$

where $k = |\mathbf{k}|$, $\hat{\mathbf{k}} = \mathbf{k}/k$, \mathbf{s} is a classical spin vector of length 1 associated with the external potential, and $\boldsymbol{\sigma}$ is the nucleon spin operator. Here, U_0 is a spin-independent potential while U_S and U_T represent a spin-dependent scalar and tensor force, respectively. After some algebra one finds

$$\begin{aligned} \sum_{\text{spins}} \frac{\langle|\mathcal{M}|^2\rangle}{2\varepsilon_1 2\varepsilon_2 2E_1 2E_2} &= \frac{G_F^2 C_A^2}{2\omega^2} \left[|U_S(k)|^2 (1 - \frac{1}{3}c_{12}) \right. \\ &- 2\text{Re}[U_T(k)U_S^*(k)] (c_1 c_2 - \frac{1}{3}c_{12}) \\ &\left. + |U_T(k)|^2 (2 - c_1 c_2 - \frac{1}{3}c_{12}) \right], \end{aligned} \quad (18)$$

where c_i ($i = 1$ or 2) is the cosine of the angle between the direction of neutrino i relative to \mathbf{k} , while c_{12} refers to the angle between the two neutrinos. We have averaged over the external spin directions \mathbf{s} with an assumed isotropic distribution.

The interaction $U_0(k)$ does not contribute because it leaves the nucleon spins unchanged. This leaves us with the scalar and tensor force $U_S(k)$ and $U_T(k)$, respectively. If the classical scatterers are substituted by the nucleons themselves only the tensor term survives because the scalar term conserves the total spin of two colliding nucleons and thus does not cause spin fluctuations [11].

Expression (18) reveals explicitly the ω^{-2} divergence of Eq. (16) which thus cannot be integrated to yield a total cross section. However, following the steps of Sect. II.A we can derive a convergent expression for the medium-induced *change* of σ_1 which denotes the total

axial-current scattering cross section for a fixed initial-state energy ε_1 . In the structure-function language it is the $d\varepsilon_2$ integral of Eq. (1) or equivalently

$$\sigma_1 = \frac{3C_A^2 G_F^2}{4\pi} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} S(\omega) (\varepsilon_1 - \omega)^2 \Theta(\varepsilon_1 - \omega). \quad (19)$$

In vacuum $\sigma_{1,\text{vac}} = (3C_A^2 G_F^2 / 4\pi) \varepsilon_1^2$ so that the medium-induced change $\delta\sigma_1 \equiv \sigma_{1,\text{med}} - \sigma_{1,\text{vac}}$ is

$$\frac{\delta\sigma_1}{\sigma_{1,\text{vac}}} = -1 + \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} S(\omega) \frac{(\varepsilon_1 - \omega)^2 \Theta(\varepsilon_1 - \omega)}{\varepsilon_1^2}. \quad (20)$$

Then we may proceed as before and replace -1 by an integral over the structure function by virtue of its normalization so that[†]

$$\frac{\delta\sigma_1}{\sigma_{1,\text{vac}}} = \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} S(\omega) \left[\frac{(\varepsilon_1 - \omega)^2 \Theta(\varepsilon_1 - \omega)}{\varepsilon_1^2} - 1 \right]. \quad (21)$$

As before, the integrand varies effectively as $S(\omega)\omega^2$ for small ω because the term linear in ω switches sign at the origin. Therefore, to lowest order we may substitute $S(\omega) \rightarrow S_{\text{brems}}(\omega)$, provided we interpret the remaining integral by its principal part.

$S_{\text{brems}}(\omega)$ is obtained by comparing Eq. (1) with Eq. (16) and using Eq. (18). After performing the $d\omega$, $d\mathbf{p}_2$, and $d\Omega_2$ integrations we arrive at

$$\begin{aligned} \frac{\delta\sigma_1}{\sigma_{1,\text{vac}}} &= \frac{2}{3} \frac{n_c}{n_B} \int \frac{d^3\mathbf{p}_1}{(2\pi)^3} \frac{d^3\mathbf{k}}{(2\pi)^3} f_1 \\ &\times \frac{|U_S(k)|^2 + 2|U_T(k)|^2}{\omega^2} \left[\frac{(\varepsilon_1 - \omega)^2 \Theta(\varepsilon_1 - \omega)}{\varepsilon_1^2} - 1 \right], \end{aligned} \quad (22)$$

where the energy transfer is $\omega = -(2\mathbf{p}_1 \cdot \mathbf{k} + \mathbf{k}^2)/2m_N$.

B. Comparison with Sawyer's Result

As mentioned in the introduction, Sawyer [12] has discussed a cross-section reduction due to the interaction of the target nucleons with bystander particles. He does not provide an immediate physical interpretation of his calculation, but we believe that in essence he has studied the same effect that is the topic of our paper, namely the scattering version of the Landau-Pomeranchuk-Migdal

effect. However, his formal approach is quite different from ours.

The optical theorem implies that calculating the total neutrino scattering cross section amounts to a calculation of the imaginary part of the neutrino forward-scattering amplitude f_0 on nucleons. Sawyer uses analyticity constraints [17] for f_0 to recognize that the total cross section should be finite order by order in a perturbative expansion in powers of the nucleon interaction potential. Further, he observes that this result holds even though individual contributions to f_0 have infrared singularities from on-shell intermediate states. One type of V^2 contribution to f_0 comes from interference between the zeroth and second order scattering amplitudes. Fig. 2 shows such a contribution which may be interpreted as a wavefunction renormalization of the incoming nucleon. Similar terms would renormalize the outgoing nucleon wave function, or provide a vertex correction. The other type of V^2 contribution to $\text{Im}(f_0)$ is given by a phase-space integral over the square of the amplitudes shown in Fig. 1. These terms correspond to the quasi-bremsstrahlung inelastic scattering process, and also diverge as discussed above. Sawyer's main point is that the divergences in these two types of terms must sum to a finite result.

Sawyer [12] has worked out several examples which illustrate this approach. Specifically, the cross-section reduction represented by his Eq. (10) is very similar to our Eq. (22). However, our Eq. (22) has not yet been averaged over initial state neutrino energies, Sawyer has used bystander nucleons to provide the potential so that his expression for the energy transfer takes account of the bystander recoil, and he has studied $\nu n \rightarrow p e^-$ scattering rather than $\nu N \rightarrow N\nu$ so that the proton-neutron mass difference appears. Further, he has used a scalar potential which is explicitly isospin dependent so that the role of our spin fluctuations is played by isospin fluctuations in his case.

The main point of agreement is the structure of the term in square brackets in Eq. (22). Both Sawyer's Eq. (10) and our Eq. (22) diverge if one considers the first term in square brackets independently from the -1 . In our derivation, the -1 effectively represents the sum rule of the nonperturbative $S(\omega)$ for which we have substituted $S_{\text{brems}}(\omega)$ after the two terms have been combined. In Sawyer's approach, the -1 corresponds to the wavefunction renormalization of the elastic scattering rate. Our interpretation of the agreement between these results is as follows.

In effect, Sawyer has calculated the perturbative approximation $S^{(2)}(\omega)$ to second order in V . Recall that the zeroth-order approximation is $S^{(0)}(\omega) = 2\pi\delta(\omega)$ because Sawyer also uses the long-wavelength limit where nucleon recoil effects are ignored. In this limit, any nonvanishing power of $S^{(2)}(\omega)$ at $\omega \neq 0$ must arise from the quasi-bremsstrahlung amplitudes of Fig. 1 so that inevitably $S^{(2)}(\omega) = S_{\text{brems}}(\omega)$ for $\omega \neq 0$. Sawyer's renormalization terms modify only the elastic channel $\omega = 0$ so that his complete result amounts to $S^{(2)}(\omega) = S_{\text{brems}}(\omega) - A\delta(\omega)$

[†]In this form one can easily see that for small ε_1 the cross section actually increases. For example, $\varepsilon_1 = 0$ leads to a vanishing vacuum cross section while in the medium it is $(3C_A^2 G_F^2 / 4\pi) \int_{-\infty}^0 d\omega \omega^2 S(\omega) / 2\pi$ or by detailed balance $(3C_A^2 G_F^2 / 4\pi) \int_0^\infty d\omega \omega^2 S(\omega) e^{-\omega/T} / 2\pi$.

where A is an infinite integral expression. Of course, $S^{(2)}(\omega)$ is highly singular and thus unphysical at $\omega = 0$ in the sense that in the neighborhood of $\omega = 0$ it does not provide a representation of the differential scattering cross section Eq. (1). However, $S^{(2)}(\omega)$ is legitimate as an integral kernel to calculate the total cross section Eq. (4). The agreement between Sawyer's and our results shows that the second-order perturbative calculation yields an expression for A such that $S^{(2)}(\omega)$ fulfills our sum rule.

In essence, then, Sawyer's calculation amounts to showing explicitly that the renormalization terms not only cancel the low- ω divergence of $S_{\text{brems}}(\omega)$, but indeed cancel it in such a way that $S^{(2)}(\omega)$ fulfills the sum rule. The renormalization terms are an explicit second-order manifestation of the information embodied in our sum rule. In our derivation, we have shown the sum rule to be a general nonperturbative property of $S(\omega)$. Therefore, once we have calculated $S_{\text{brems}}(\omega)$ we can handle its low- ω divergence either by an abstract application of the sum rule, or by an explicit ansatz for the physical behavior of the true $S(\omega)$ near $\omega = 0$. Either way we do not need to calculate the renormalization terms explicitly.

Although Sawyer's and our approaches are equivalent in the low-density limit, they are not equivalent when one considers the high-density case. There, a perturbative expansion makes no sense as higher-order terms exceed the lower-order ones. However, by making use of the sum rule, and exploiting the physical insight that $S(\omega)$ should have a width of order Γ_σ [11] and possess a hard bremsstrahlung tail for $\omega \gg \Gamma_\sigma$, we have the basis for a reasonable model of the high-density regime.

To summarize, our derivation is based on representing interaction rates by virtue of current correlators which allow for a direct transition to the classical limit. Therefore, our approach allows for an intuitive interpretation of the cross-section reduction as a temporal spin-averaging effect. Moreover, because we know on general grounds that the sum rule Eq. (6) must be fulfilled, we do not need to worry about a calculation of the various infinite second-order corrections to the elastic scattering rate. In our derivation, the only required Feynman-graph evaluation is that of the “medium excitation term” of Fig. 1. Finally, our derivation allows for a clear and physical identification of the dimensionless parameter γ_σ which defines the perturbative expansion. Sawyer's technique, on the other hand, represents a more familiar methodology if one approaches the problem as a perturbative expansion in powers of V .

IV. ENERGY TRANSFER

As stressed in Ref. [10], the most important effect of nucleon spin fluctuations may be that they allow for a new mode of energy transfer by the quasi-bremsstrahlung process shown in Fig. 1. The relevant figure of merit is the average energy transfer per collision $\langle \Delta\varepsilon \rangle_{\text{brems}}$ or

$$\frac{\int_0^\infty d\varepsilon_1 \varepsilon_1^2 e^{-\varepsilon_1/T_\nu} \int_0^\infty d\varepsilon_2 \varepsilon_2^2 (\varepsilon_2 - \varepsilon_1) S(\varepsilon_1 - \varepsilon_2)}{\int_0^\infty d\varepsilon_1 \varepsilon_1^2 e^{-\varepsilon_1/T_\nu} \int_0^\infty d\varepsilon_2 \varepsilon_2^2 S(\varepsilon_1 - \varepsilon_2)}. \quad (23)$$

Here, T_ν is the temperature of the neutrinos which are assumed to follow a Maxwell-Boltzmann distribution while the nucleons are characterized by T . In Ref. [10] this expression was evaluated numerically on the basis of the Lorentzian ansatz for the structure function.

However, in the dilute-medium limit one can also derive an explicit expression. We first note that in the numerator and denominator one can each perform one integration explicitly so that

$$\frac{\langle \Delta\varepsilon \rangle_{\text{brems}}}{T} = \frac{\int_0^\infty dx \tilde{S}(x) x F_\beta(x) (e^{-x} - e^{-\beta x})}{\int_0^\infty dx \tilde{S}(x) F_\beta(x) (e^{-x} + e^{-\beta x})} \quad (24)$$

with $F_\beta(x) \equiv 1 + \frac{1}{2}\beta x + \frac{1}{12}\beta^2 x^2$ and $\beta \equiv T/T_\nu$.

In the dilute-medium limit we may use to lowest order $\tilde{S}(x) \rightarrow \tilde{S}_{\text{brems}}(x)$ in the numerator, while in the denominator $\tilde{S}(x) \rightarrow 2\pi\delta(x)$ because the medium-induced change of the cross section is itself of order γ_σ . With the representation Eq. (9) we find

$$\langle \Delta\varepsilon \rangle_{\text{brems}} = \Gamma_\sigma \int_0^\infty \frac{dx}{2\pi} s(x) \frac{F_\beta(x)}{x} (e^{-x} - e^{-\beta x}). \quad (25)$$

For the classical limit of hard collisions where $s(x) = 1$ this is

$$\langle \Delta\varepsilon \rangle_{\text{brems}} = \Gamma_\sigma \frac{-7 + 6\beta + \beta^2 + 12\ln\beta}{24\pi}. \quad (26)$$

This is to be compared with the average energy transfer by nucleon recoils, $\langle \Delta\varepsilon \rangle_{\text{recoil}} = 30(\beta-1)\beta^{-2}T^2/m_N$ [18]. Therefore, the ratio between the two is

$$\begin{aligned} \frac{\langle \Delta\varepsilon \rangle_{\text{brems}}}{\langle \Delta\varepsilon \rangle_{\text{recoil}}} &= \frac{\Gamma_\sigma m_N}{T^2} \beta^2 \frac{-7 + 6\beta + \beta^2 + 12\ln\beta}{720\pi(\beta-1)} \\ &= \frac{\Gamma_\sigma m_N}{T^2} \left(\frac{1}{36\pi} + \frac{7(\beta-1)}{144\pi} + \dots \right). \end{aligned} \quad (27)$$

Therefore, the importance of the “inelastic” mode of energy transfer exceeds that of recoils if $\gamma_\sigma > 36\pi T/m_N$.

We note that the quasi-bremsstrahlung process of Fig. 1 has a standard counterpart where neutrino pairs are absorbed or emitted. We define a rate of energy transfer in this channel, normalized to the average neutrino scattering rate in analogy to the above discussion. By virtue of Ref. [5] the result can be expressed like Eq. (25) with $F_\beta(x) = \beta^5 x^5/1440$. The efficiency of energy transfer relative to recoil effects is

$$\frac{\langle \Delta\varepsilon \rangle_{\text{pair}}}{\langle \Delta\varepsilon \rangle_{\text{recoil}}} = \frac{\Gamma_\sigma m_N}{T^2} \beta^2 \frac{(\beta^2 + 1)(\beta + 1)}{3600\pi}. \quad (28)$$

Therefore, the quasi-bremsstrahlung process of Fig. 1 is approximately a factor of 25 more important than pair processes.

V. DISCUSSION AND SUMMARY

To summarize, we have studied the neutrino-nucleon scattering cross section taking into account nucleon spin fluctuations. The effect of random spin fluctuations is to reduce the cross section in a manner similar to the LPM reduction of photon bremsstrahlung by multiple-scattering effects. We have derived perturbative results in terms of a lowest-order spin-density structure function, and also in terms of the squared matrix element of neutrino-nucleon scattering in the presence of bystander nucleons or more general external spin-dependent potentials. In this form, our result agrees with Sawyer's [12] related finding. The low-density limit is unique unless there are unexpectedly strong spin-spin correlations.

While we have focussed on neutral-current processes, similar conclusions would obtain for charged-current collisions as stressed in Refs. [9,12].

The explicit low-density results are theoretically interesting, but their practical significance is limited. It is obvious from Fig. 4 that a plausible extrapolation into the high-density regime vastly differs from the perturbative result for $\gamma_\sigma = \Gamma_\sigma/T \gtrsim 1$ which implies that the perturbative result cannot be trusted for densities greater than a few percent nuclear. While $\Gamma_{\sigma,\text{OPE}}$ overestimates the true Γ_σ at nuclear density, in a SN core one has values for the true γ_σ of order a few, perhaps as large as 10 [10,11]. Therefore, the neutrino opacities in the inner SN core cannot be treated by perturbation theory alone. Near the neutrino sphere, corresponding to $\gamma_\sigma = \mathcal{O}(1)$, a perturbative treatment is roughly justified, but the cross-section reduction is small (a few percent) and thus not overly significant.

Near the neutrino sphere, the most important practical consequence of nucleon spin fluctuations is likely to be the inelastic or quasi-bremsstrahlung mode of energy transfer. With Eq. (27) and taking $T = 5\text{ MeV}$ as a typical neutrino-sphere temperature it is found to compete with standard recoils for $\gamma_\sigma \gtrsim 0.5$. As this value is representative for conditions around the neutrino sphere, we confirm that the inelastic mode of energy transfer is about as efficient as recoils and thus may be important for the formation of neutrino spectra [10].

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